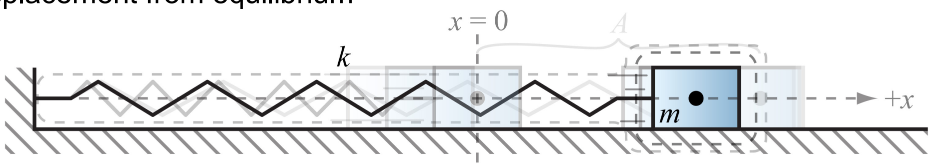
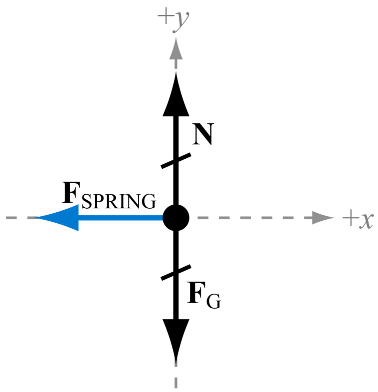
Ingredients for simple harmonic motion (SHM)

Mass on a spring

x – displacement from equilibrium





Spring force
Direction: Restoring
Magnitude: ∝ Displacement

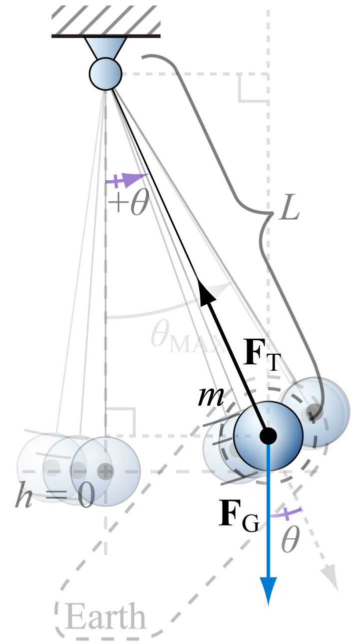
$$a_x = \frac{\Sigma F_x}{m}$$

$$a_x = \frac{-k|\Delta x|}{m}$$

$$a_x = \frac{-kx}{m}$$

$$a_x = -\frac{k}{m}x$$

Ideal pendulum



θ – angular displacement from equilibrium

Gravitational torque
Direction: Restoring
Magnitude: Approx. ∝ Angular displacement

$$\alpha = \frac{\Sigma\tau}{I}$$

$$\alpha = \frac{-mgL\sin\theta}{mL^2}$$

$$\alpha = -\frac{g}{L}\sin\theta$$

For small angles,

$$\alpha \approx -\frac{g}{L}\theta$$

$$\omega^2 = \frac{k}{m}$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \delta\right)$$

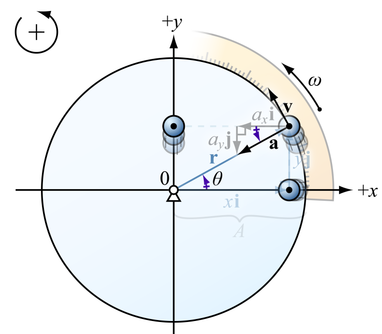
$$= A\cos\left(\frac{2\pi}{T}t + \delta\right)$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{1}{f} = T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$\Sigma_{\mathrm{MASS\ \&\ SPRING}} ME = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$

ω – angular velocity of UCM that completes one lap in the same duration of time that SHM of interest completes one cycle of oscillation



A – amplitude (maximum linear or angular distance from equilibrium)

T – period (repetition time)
When magnitude of restoring net force (torque) is ∝ (angular) displacement, T is independent of A

f – frequency (oscillations per unit time)

$$\omega^2 = \frac{g}{L}$$

$$\theta(t) = \theta_{\mathrm{MAX}}\cos\left(\sqrt{\frac{g}{L}}\,t + \delta\right)$$

$$= \theta_{\mathrm{MAX}}\cos\left(\frac{2\pi}{T}t + \delta\right)$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{1}{f} = T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$\Sigma_{\mathrm{BOB\ \&\ \oplus}} ME = \frac{1}{2}mv_{\mathrm{TAN}}^2 + mgL(1-\cos\theta)$$

$$\Sigma_{\mathrm{BOB\ \&\ \oplus}} ME \approx \frac{1}{2}mv_{\mathrm{TAN}}^2 + \frac{mg}{2L}(L\theta)^2$$